Dynamic Signature Verification System Based on One Real Signature

Moises Diaz, *Member, IEEE*, Andreas Fischer, *Member, IEEE*, Miguel A. Ferrer, and Réjean Plamondon, *Fellow, IEEE*

***Abstract*—The dynamic signature is a biometric trait widely used and accepted for verifying a person's identity. Current automatic signature-based biometric systems typically require five, ten, or even more specimens of a person's signature to learn intrapersonal variability sufficient to provide an accurate verification of the individual's identity. To mitigate this drawback, this paper proposes a procedure for training with only a single reference signature. Our strategy consists of duplicating the given signature a number of times and training an automatic signature verifier with each of the resulting signatures. The duplication scheme is based on a sigma lognormal decomposition of the reference signature. Two methods are presented to create human-like duplicated signatures: the first varies the strokes' lognormal parameters (stroke-wise) whereas the second modifies their virtual target points (target-wise). A challenging benchmark, assessed with multiple state-of-the-art automatic signature verifiers and multiple databases, proves the robustness of the system. Experimental results suggest that our system, with a single reference signature, is capable of achieving a similar performance to standard verifiers trained with up to five signature specimens.**

***Index Terms*—Duplicated signatures, dynamic signature verification, kinematic theory of rapid human movements, single reference signature system (SRSS).**

I. Introduction

VERIFYING the identity of people through their signatures is an important goal in the field of biometrics [1], [2]. While much more stable traits, such as the fingerprint and iris are often used because of their high performances, handwriting signatures are still being used and researched. The signature's cultural acceptance for personal authentication and its presence in wills, contracts and other important documents over centuries makes it a worthwhile trait to justify the efforts of the research community and industry.

The execution of a signature depends on a very complex system, strongly influenced by behavioral and social conditions. As a result, two repetitions of a signature from the same writer never have an identical appearance. This effect is known as intrapersonal variability. Consequently, many systems have different effective error rates for verifying the authenticity of a signature, depending on the training conditions. One of the main limitations of current systems is the number of training signatures required to learn the unpredictable level of intrapersonal variability. The more signatures enrolled during training, the better the expected test performance.

Nevertheless, in a real situation, it is often impractical to obtain many signature samples from a client, for example, in the context of banking applications. Therefore, this paper explores strategies to design an automatic signature verification system using only one real reference signature per enrolled signer. Dynamic signature acquisition is chosen because it is a natural modality for the user.

In this context, signature variability is rarely derived from a single sample but this paper shows that it can be usefully achieved by means of a human behavioral model of signature kinematics. One of the most mature models for human movement analysis is the kinematic theory of rapid movements [3]–[5]. This theory has demonstrated its effectiveness in the development of tools for learning handwriting in children [6], detection of problems related to brain strokes [7], and especially in signature verification [8]–[15].

In particular, the sigma-lognormal model [9] analytically decomposes the complex movement into a linear combination of lognormal strokes. Based on lognormal parameters, it is possible to obtain a robust mathematical framework, able to exploit the intrapersonal variability of a signature from both a neuroscience and a computational point of view. This paper uses these perspectives for the design of a single reference signature system (SRSS), which duplicates signatures derived from only a single, real reference specimen for synthesizing human-like intrapersonal variability, related to both the signature shape and its kinematic properties.

Manuscript received April 18, 2016; revised August 28, 2016; accepted November 14, 2016. Date of publication December 6, 2016; date of current version November 15, 2017. This work was supported in part by the Spanish Government's MCINN under Grant TEC2016-77791-C4-1-R, and in part by the European Union FEDER Program/Funds. The work of M. Diaz was supported by the Ph.D. Fellowship from the ULPGC. The work of A. Fischer was supported by the Swiss National Science Foundation Fellowship under Project P300P2-151279. The work of R. Plamondon was supported by the NSERC CANADA under Grant RGPIN-2015-06409. This paper was recommended by Associate Editor B. Sick.

M. Diaz and M. A. Ferrer are with IDeTIC Institute, Universidad de Las Palmas de Gran Canaria, 35017 Las Palmas, Spain (e-mail: mdiaz@idetic.eu; mferrer@idetic.eu).

A. Fischer is with DIUF Department, University of Fribourg, 1700 Fribourg, Switzerland, and also with iCoSys Institute, University of Applied Sciences and Arts Western, 1700 Fribourg, Switzerland (e-mail: andreas.fischer@unifr.ch; andreas.fischer@hefr.ch).

R. Plamondon is with the École Polytechnique de Montréal, Laboratoire Scribens, Montreal, QC H3T 1J4, Canada (e-mail: rejean.plamondon@polymtl.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCYB.2016.2630419

A. Related Work

This paper belongs to the active research field of automatic handwritten signature verification. Many techniques have been explored to solve the problem of signature verification, the vast majority of which have been discussed in comprehensive surveys [16]–[21]. Although most methods have achieved reasonable levels of performance, there are few examples in the literature of the use of an SRSS for verification purposes. However, inspired by the forensic handwriting expert's task of comparing a single reference signature and a questioned specimen, the ICDAR 2009 Signature Verification Competition [22] proposed to participants, who had to use a specified system, the solving of the problem of a single reference signature with a specific signature database. In addition to not describing either the methodology to participants or the systems to the scientific community, according to the competition rules, only the skilled forgery experiment was considered. However, in this paper, we describe the design of an SRSS, which can be used for standard automatic signature verifiers for both random and skilled forgery tests and with multiple databases.

One of the first contributions on using an SRSS for automatic verification was reported in [23]. Galbally *et al.* [23] followed the strategy of duplicating the reference signature to enlarge the training set. To duplicate signatures, the work discussed a three step architecture: 1) the addition of low-pass noise to the original trajectory and a pressure signal; 2) the modification in the duration of the signatures by an expansion or contraction of the signals; and 3) affine and geometrical distortions to the shape of the duplicates. The results achieved were around three times better for the random forgery tests and two times better for the skilled forgery tests as compared to their own baseline for the reference signature alone. It is also noteworthy that their experiments were conducted with a specific online signature database and a hidden Markov model (HMM) classifier.

A second proposal by the same authors can be found in [10]. Their strategy consisted in duplicating many times the reference signature by using the kinematic theory of rapid human movement. New specimens were obtained by introducing three sources of Gaussian distortion into the parameterized signatures and then resynthesizing again. Despite promising results from a performance-based perspective, the method was not directly compared to [23] because of the use of another different database as a different verifier. Additionally, more experiments were required to prove the robustness of the method.

In the offline domain, we have found a few systems which duplicate signatures as a solution to modeling the spatial intrapersonal variability when the static signatures are generated from online specimens. As a consequence, in [24], two signatures aligned with dynamic time warping were used as the basis of generating duplicated specimens. A similar situation was studied in [25]. In this case, Ferrer *et al.* [26] proposed a cognitive model to duplicate the trajectory and the use of an ink deposition model to represent the image-based duplicated signatures. Nevertheless, none of these contributions duplicated the dynamic properties of the signatures. Therefore, the verification was carried out through offline signature verifiers so as to achieve the improvements in the performance.

B. This Paper

The main contribution of this paper is the design of an SRSS for verification purposes. We use a biological neuromuscular model to duplicate online signatures as a means to deal with intrapersonal variability. As such, not only is the trajectory of the signatures duplicated, but also their dynamic properties such as velocity and acceleration.

The procedure proposed in this paper considers a biological and neuromuscular model to reproduce signatures and to build a robust SRSS. Thus, we complete the exploratory work reported in [10]. Two procedures for generating duplicated signatures are suggested here: 1) the former consists of modifying all the sigma-lognormal parameters by which a single stroke is redefined and duplicated and 2) the latter modifies the strokes by perturbation of the target points of their action plan with a cognitive inspired model [25] and rebuilding the signature. Each approach, called the stroke-wise (SW) and the target-wise (TW) method, respectively, has the property of generating human-like duplicate signatures with a realistic intrapersonal variability. The proposed system which fits the business requirement of using only a single available enrolled signature is assessed with multiple classifiers and databases.

The outline of this paper is as follows. The following section contains a brief description of the neuromuscular representation of the signatures with the sigma-lognormal model. Section III presents the proposed generation methods we use to produce duplicated signatures (SW and TW methods). Databases and automatic signature verifiers are described in Section IV. The setup of the SRSS is presented in Section V. Experimental results and a comparison with the state-of-the-art are discussed in Section VI. Finally, concluding remarks are drawn in Section VII.

II. Sigma-Lognormal Model for Signatures

The sigma-lognormal model is the design basis of the SRSS.[1] This section details the use of this biological model [27] to calculate the intrapersonal variability using the generated duplicated signatures. Fig. 1 shows an example of signature reconstruction by using this model.

A. Signature Preprocessing

Standard signal preprocessing was carried out for dynamic signatures captured by any device such as LCD touch pad, Wacom, handheld, etc. The preprocessing prepared the signatures for subsequent extraction of the sigma-lognormal model parameters. This consisted of three consecutive steps applied to each component trajectory.

1) *Trajectory Resampling:* Each component was resampled at 200 Hz, which was the suggested sample rate for extracting the sigma-lognormal model data. As such, the

[1] We use Script-Studio software with dynamic signatures to extract the lognormal parameters. This software is shared after signing a license agreement.

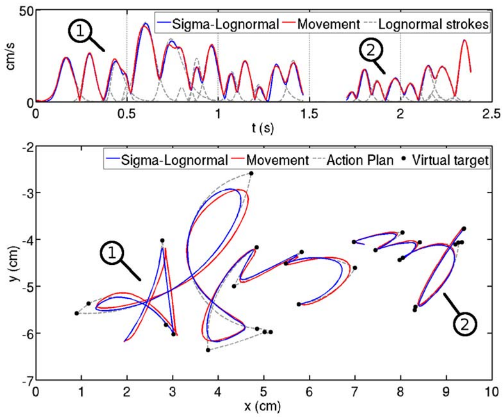

Fig. 1. Example of real and reconstructed signature. *Stroke* refers to a neuromuscular command to execute an elementary movement, which is drawn in gray dotted lines. The term *component* is used to describe the number of pen-downs, two in this example. *Trajectory* involves the whole movement which can comprise several components, each component being generally made up of many strokes.

original time sequence was artificially substituted with a new one sampled at 200 Hz and based on cubic spline interpolation.

2) *Trajectory Smoothing:* Next, a Chebyshev filter was applied to the interpolated trajectory. The filter enhanced the signals, therefore removing the particular noise often introduced by the capturing device.
3) *Trajectory Enlarging:* For 200 ms at 200 Hz the initial and the final sampling points were repeated both for the horizontal and vertical signals independently. This third step introduced a null velocity during the first and last 200 ms and led to an improved extraction of the first and the last stroke parameters.

B. Sigma-Lognormal Parameter Extraction

The kinematic theory of rapid human movements describes a movement as resulting from the controlled activation of the impulse response of a neuromuscular system, which is modeled through the vector summation of lognormal functions [5], [8], [9], [28]. Each component of the trajectory is analyzed individually. Note that in this context the term component refers to the trajectory between the beginning and the end of a pen-down movement. A component is usually composed of several elementary strokes hidden in the signal.

Each stroke $\{P_i\}_{i=1}^{i=n}$ is represented by one parameterized lognormal, n being the total number of strokes made during the execution of one component. The theory assumes that each individual impulse during the signature starts by executing the ith lognormal movement at time t_{0_i} by inputting a command D_i into the neuromuscular system. The execution of this movement depends on the timing properties of the neuromuscular network activated, which is represented by the parameters μ_i the log-time delay and σ_i, the log-response time. It is also assumed that the movement of a single stroke occurs along a pivot with respect to a starting angle θ_{s_i} and an ending angle θ_{e_i}. In total, each stroke is described in 2-D space by six sigma-lognormal parameters: $P_i = (D_i, t_{0_i}, \mu_i, \sigma_i, \theta_{s_i}, \theta_{e_i})$.

The velocity of the complete handwriting movement is considered as the vector summation of the individual stroke velocities $\vec{v}(t) = \sum_{i=1}^{n} \vec{v}_i(t)$, where the magnitude and direction of each stroke is described as

$$|\vec{v}_i(t)| = \frac{D_i}{\sqrt{2\pi}\sigma_i\left(t - t_{0_i}\right)} \exp\left(-\frac{\left(\ln\left(t - t_{0_i}\right) - \mu_i\right)^2}{2\sigma_i^2}\right) \quad (1)$$

$$\phi_i(t) = \theta_{s_i} + \frac{\theta_{e_i} - \theta_{s_i}}{D_i} \int_0^t |\vec{v}_i(\tau)| d\tau. \quad (2)$$

A central part of the framework is the fully automatic extraction of the stroke sequence $\mathbf{P} = \{P_1, \ldots, P_i, \ldots, P_n\}$ from observed pen tip velocity profiles. The algorithm [9] is based on two main steps.

In the first step, it localizes the strokes P_i using the original velocity $\vec{v}(t)$. A local maximum is identified in the speed profile along with neighboring inflexion points and minima. To identify a stroke, the maximum speed and the area under the curve have to be greater than a certain threshold. The second step extracts the analytical parameters of each identified stroke on the basis of zero crossings of the first and second derivatives of the lognormal equation. The result of this Robust XZERO (RX_0) estimator is further improved with nonlinear least squares curve fitting. These two steps are repeated until the quality of the stroke sequence cannot be further improved. The quality of the extraction process is estimated with respect to the squared Euclidean distance between the original velocity $\vec{v}_o(t)$ and the reconstructed velocity $\vec{v}_r(t)$ expressed as the signal-to-noise ratio (SNR)

$$\text{SNR} = 10 \log\left(\frac{\int_{t_s}^{t_e} |\vec{v}_o(\tau)|^2 d\tau}{\int_{t_s}^{t_e} |\vec{v}_o(\tau) - \vec{v}_r(\tau)|^2 d\tau}\right). \quad (3)$$

The time t_s is the start time and t_e is the end time of the trajectory. High SNR values indicate high reconstruction quality of the speed profile. We refer the reader to [9] for more details on the neuromuscular representation framework.

C. Signature Reconstruction

The trajectory of the analytical signature is obtained component by component. It can be reconstructed by the following equations:

$$v_x(t) = \sum_{i=1}^{n} |\vec{v}_i(t)| \cos(\phi_i(t)), v_y(t) = \sum_{i=1}^{n} |\vec{v}_i(t)| \sin(\phi_i(t)) \quad (4)$$

$$x(t) = \int_0^t v_x(\tau) d\tau, y(t) = \int_0^t v_y(\tau) d\tau. \quad (5)$$

Because the trajectory of each component is extended during preprocessing to improve the parameter extraction, the first and final 200 ms are ignored for both $x(t)$ and $y(t)$.

Once all trajectory components are processed, to build the whole signature, each component is joined, starting at the same time and position as in the original signature. This guarantees the coincidence between the initial point of the original and the reconstructed signatures.

III. Generation of Duplicated Signatures

Two methods are proposed for duplicating the reference signature: 1) an SW sigma-lognormal parameter distortion method and 2) a TW sigma-lognormal action plan distortion method. In each method, the lognormal signature parameters are modified—$P_i \rightarrow \widehat{P}_i = (\widehat{D}_i, \widehat{t}_{0_i}, \widehat{\mu}_i, \widehat{\sigma}_i, \widehat{\theta}_{s_i}, \widehat{\theta}_{e_i})$—to mimic from only one real specimen the human signature intrapersonal variability.

One of the major advantages of the sigma-lognormal model is its use for the neuromuscular decomposition of the movement into elementary strokes and the recovery of the initial action plan. This enables us to generate new trajectories by keeping the intrapersonal variability at stroke level, instead of altering the observed trajectory. In this paper, the number of strokes of the original signature is not modified, only the parameters that define a stroke.

A. Method 1: Stroke-Wise Distortion Method

In this method, the intrapersonal variability is artificially introduced by changing the sigma-lognormal parameters stroke by stroke. Three sources of variability are employed in this model: 1) temporal; 2) spatial; and 3) neuromuscular. All the perturbations are based on Gaussian noise.

1) The neuromuscular intrapersonal variability is achieved by distorting the parameters related to the neuromuscular execution of the stroke

$$\widehat{\mu}_i = \mathcal{N}\Big(\mu_i; \big(\mu_i \cdot d_\mu\big)^2\Big) \tag{6}$$

$$\widehat{\sigma}_i = \mathcal{N}\Big(\sigma_i; (\sigma_i \cdot d_\sigma)^2\Big). \tag{7}$$

2) The intrapersonal variability regarding the motor command time occurrence of each stroke is generated by

$$\widehat{t}_{0_i} = t_{0_i} + \mathcal{N}\Big(0; \big(d_{t_0}\big)^2\Big). \tag{8}$$

3) To represent the geometrical intrapersonal variability in each stroke, a deformation is applied to the magnitude and stroke direction as follows:

$$\widehat{D}_i = \mathcal{N}\Big(D_i; (D_i \cdot d_D)^2\Big) \tag{9}$$

$$\widehat{\theta}_{s_i} = \theta_{s_i} + \mathcal{N}\Big(0; \big(d_{\theta_s}\big)^2\Big) \tag{10}$$

$$\widehat{\theta}_{e_i} = \theta_{e_i} + \mathcal{N}\Big(0; \big(d_{\theta_e}\big)^2\Big). \tag{11}$$

As extreme values of the normal distribution could distort the human-like appearance of the duplicated specimens, the randomly generated values are clipped in the range of twice the standard deviation which comprises 95 % of the distribution.

Once a new sequence of strokes parameters $\widehat{P}_i$ is obtained, the duplicated signature is reconstructed according to the $\Sigma\Lambda$ (1), (2), (4), and (5). Each duplicated component starts at the same position as the original one.

A similar method was proposed in [14] in the context of fully synthetic generation of flourish like signatures. In this paper, the SW distortion method allows us to generate duplicated specimens from a real signature that contains a flourish and text. These duplicated signatures are used to train an automatic signature verifier along with the single reference one. The verifier is tested with real signatures, which is a difference with respect to [14] since training and testing is performed only with synthetic signatures in that study. It is worth mentioning that the main difficulty in this paper is the incorporation of duplicated signatures into real biometric signature verification processes since the duplicates have to be generated by fitting the peculiarities of the intrapersonal variability of real people.

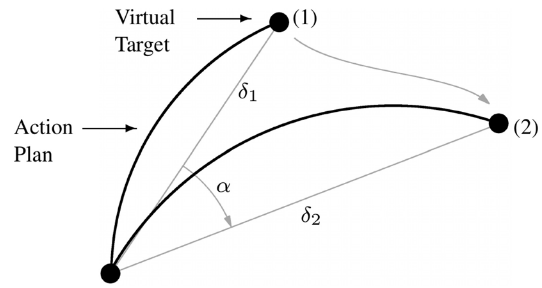

Fig. 2. Computation of the scale factor and rotation angle when a virtual target point is sinusoidally translated.

B. Method 2: Target-Wise Distortion Method

An advantage of the $\Sigma\Lambda$ model is the extraction of parameters which allows us to recover the virtual target points of the action plan used to generate a given trajectory [19]. Virtual targets are defined as the end points of the strokes when executed in isolation. A signature is the result of multiple overlapped strokes. Therefore, in most cases, the writing instrument does not reach the virtual points, except for the last stroke, which ends at its corresponding virtual target.

The model introduced in [25] suggests that the sinusoidal mapping of the target points generates a human-like variability for obtaining a new signature sample. This concept is combined with the lognormal formulation by applying such a sinusoidal transformation to the virtual target points as follows:

$$\widehat{x}_{\mathrm{VT}} = x_{\mathrm{VT}} + A_x \sin(\omega_x x_{\mathrm{VT}} + \phi_x) \tag{12}$$

$$\widehat{y}_{\mathrm{VT}} = y_{\mathrm{VT}} + A_y \sin\big(\omega_y y_{\mathrm{VT}} + \phi_y\big) \tag{13}$$

where the real virtual target point coordinates are denoted by $(x_{\mathrm{VT}}, y_{\mathrm{VT}})$ and the duplicated ones as $(\widehat{x}_{\mathrm{VT}}, \widehat{y}_{\mathrm{VT}})$. (A_x, A_y) refers to the amplitude of the sinusoid, (ω_x, ω_y) to the oscillation frequency, and (ϕ_x, ϕ_y) to the phase. Fig. 2 illustrates the geometrical consequence in the action plan when a virtual target is moved. Accordingly, D_i, θ_{s_i}, and θ_{e_i} are modified by using $\lambda = \delta_2/\delta_1$ and the rotation angle α, as depicted in Fig. 2

$$\widehat{D}_i = D_i \cdot \lambda \tag{14}$$

$$\widehat{\theta}_{s_i} = \theta_{s_i} + \alpha \tag{15}$$

$$\widehat{\theta}_{e_i} = \theta_{e_i} + \alpha. \tag{16}$$

The sigma-lognormal parameters related to the neuromuscular intrapersonal variability and the motor command time occurrence of every stroke were modified in accordance with the normal distribution defined in (6)–(8). All parameters were modified anew in this second method.

As the initial point of every component changes its position because of the pen-up to pen-down transition, this procedure

TABLE I
SNR-BASED EVALUATION FOR GENUINE, FORGERIES AND WHOLE SIGNATURE DATABASE

		Genuine		Forgeries		Whole DB	
Database	# Sign.	Mean	SD	Mean	SD	Mean	SD
SUSIG-Visual	2820	20.03	2.00	19.55	3.01	19.87	2.40
SUSIG-Blind	1700	20.15	2.02	17.49	4.49	18.77	3.76
SVC-Task1	1600	18.82	3.70	19.77	3.25	19.30	3.51
SVC-Task2	1600	18.93	3.50	19.65	3.36	19.30	3.51
MCYT-100	5000	20.62	2.27	20.66	3.62	20.64	3.02
SG-NOTE	500	20.53	2.25	-	-	20.53	2.25

introduces a certain variability to the component trajectory positioning. Finally, this algorithm also introduces a realistic variability to the original skew of the whole signature with respect to ascenders and descenders. The parameters regarding with this deformation were previously optimized in [25].

IV. DATABASES AND VERIFIERS

A. Databases

We have studied the SRSS on six publicly available online signature databases that are widely used in the literature. The main differences among these databases were the acquisition protocol, the geographical location and registering device. In the following, the databases used in this paper are briefly described.

1) SUSIG-Visual Subcorpus [29]: It is considered for its wide acceptance in many research papers. This database contains 94 users with 20 genuine signatures, acquired in two sessions, and ten skilled forgery signatures. This subcorpus was collected with an LCD touch device.

2) SUSIG-Blind Subcorpus [29]: It consists of 88 users with 8 or 10 genuine repetitions and 10 forged signatures per user. The volunteers could not see the signature trajectory as visual feedback during the acquisition process was denied.

3) SVC-Task1 Subcorpus [30]: It includes both Chinese and English signatures, each captured by a WACOM tablet. This subcorpus is composed of 40 users with 20 genuine and 20 forged signatures per user. Because only the dynamic trajectory is provided, this subcorpus is not popular among the scientific community.

4) SVC-Task2 Subcorpus [30]: It is one of the most widely used corpuses because, apart from the sampled trajectory, this subcorpus also provides the pressure and pen-orientation signals. It is also composed of 40 users with the same number of genuine and forged signatures as in Task1.

5) MCYT100 Subcorpus: It is a part of the full MCYT database [31] and was captured by a WACOM tablet. It contains 100 users with 25 genuine signatures acquired in two sessions and 25 skilled forgeries.

6) SG-NOTE Database [32]: This is at present one of the few publicly available mobile signature databases. This corpus, captured with a Samsung Galaxy Note mobile phone, is composed of 25 users with 20 genuine signatures collected in two sessions.

All signatures were initially reconstructed in the sigma-lognormal domain. Table I shows the quality of the signature reconstruction in terms of the SNR for each database. Previous studies [33] have shown that an SNR greater than 15 dB is sufficient for reconstructing rapid human movements. As the results show, the reconstructed signatures can conveniently represent their original version; these reconstructed signatures are the input to the SRSS for its performance-based evaluation.

In order to unify all database conditions, we have omitted the pen-up components. This does not affect the validity of the experimental assessment as the kinematics of the pen-downs and pen-ups are similar. Moreover, it enables us to work in a more realistic domain since the current handheld devices do not register pen-ups.

B. Automatic Signature Verifiers

Three different dynamic automatic signature verifiers were used during the experiments. They were based on completely different features and matchers. These systems allowed us to study the impact of our method across different technologies belonging to the current state-of-the-art.

1) System A (DTW-Based Verifier [10], [34]): Only the trajectory signals and their first and second derivatives were used to build the feature vector per component. The final feature matrix for a signature was obtained by concatenating all feature vectors and computing the z-score. A standard version of the DTW was configured to optimize the Euclidean distance with three local transitions. The search space was reduced by a Sakoe–Chiba band [35] with a width of 10 %. The relationship of a questioned signature q to the signer model was then quantified in $s_{\mathcal{R}}(q)$ as the minimum distance between this signature and the model: $s_{\mathcal{R}}(q) = \text{argmin}_{r\in\mathcal{R}}\,[\text{DTW}(q, r)]$, where $\mathcal{R}$ includes all signatures in the training set. Then, a two-stage score normalization was carried out to compute the final score. While the warping path length $|p|$ of the minimum DTW distance was used to detect weak forgeries in a first stage, a weighted factor $\mu_{\mathcal{R}}$ calculated by the average DTW distance among the signatures used to train, copes with more skilled forgeries in a second stage.

2) System B (Manhattan Distance-Based Verifier [36]): Since this histogram-based verifier is particularly convenient for handheld devices, at least according to the reported promising results with a proprietary mobile signature database, we have implemented a version following the configuration described in [36]. We took into account the dynamic signatures under three considerations: 1) resampling; 2) concatenation of the components; and 3) addition of extra histogram features. To use a unique configuration of this verifier, all of its parameters were optimized for the SUSIG-Visual subcorpus. The same verifier was used for the other databases without any modification. The thresholds in the algorithm were set to the following values: $\beta = 0.1; \epsilon_{\text{rel}} = 0.5; \epsilon_{\text{abs}} = 0.5$ and the weights of the histograms R and $\Phi - \Phi^{d(1,2)}$ were increased three times with respect to the other histograms, following the same nomenclature as in [36].

3) System C (HMM-Based Verifier [37]): Using the same features as for the DTW-based verifier, we have implemented an HMM verifier as proposed in [37]. System parameters include the number of HMM states and the number of

TABLE II
SUSIG-VISUAL EER (%) RESULTS TRAINING WITH THE FIRST SIGNATURE PLUS DUPLICATES; SW DISTORTION METHOD AND DTW-BASED VERIFIER

Real	Dup.	Random Forgery Test								Skilled Forgery Test							
		$d_D, d_{\theta_s}, d_{\theta_e}$								$d_D, d_{\theta_s}, d_{\theta_e}$							
		0	0.005	0.015	0.025	0.05	0.1	0.2	0.4	0	0.005	0.015	0.025	0.05	0.1	0.2	0.4
1	0	8.09	8.09	8.09	8.09	8.09	8.09	8.09	8.09	15.53	15.53	15.53	15.53	15.53	15.53	15.53	15.53
1	2	3.28	2.94	3.36	2.85	2.77	2.26	2.60	4.30	10.85	10.74	11.38	10.21	8.94	9.26	8.19	8.62
1	4	2.38	2.26	2.60	3.06	2.17	2.34	2.94	3.91	9.47	10.85	10.96	8.94	8.72	8.83	8.19	8.40
1	8	2.51	2.04	2.60	2.81	2.38	2.34	2.72	3.87	8.83	9.36	9.47	8.72	8.62	8.62	8.09	8.30
1	16	2.68	2.51	2.77	2.51	2.55	2.13	2.13	3.11	9.68	9.68	10.00	9.36	8.83	8.19	7.34	7.98
1	32	3.11	2.89	2.94	2.51	2.77	**2.13**	1.49	3.11	10.53	10.43	10.21	10.21	9.47	**7.45**	7.55	7.87
1	64	3.15	3.06	3.11	3.06	2.68	2.43	1.53	3.23	10.85	10.85	10.96	10.64	9.68	7.98	7.43	7.55
1	128	3.23	3.06	3.28	3.32	2.68	2.55	1.83	3.28	12.13	11.70	11.81	11.91	10.32	8.83	7.34	7.34
1	256	3.11	3.11	3.15	3.06	2.85	2.77	1.96	3.23	12.34	12.45	12.23	12.66	11.49	8.94	7.45	7.34

Gaussian mixtures per state. We considered $\alpha \cdot L_{\mathcal{R}}$ HMM states in a linear topology, where $0 < \alpha < 1$ and $L_{\mathcal{R}}$ was the average number of sampling points of the enrolled reference signatures. Again, the system parameters were optimized on the SUSIG-Visual subcorpus and were used for the rest of the databases without any modification. In particular, we set $\alpha = 0.04$ and used 11 Gaussian mixtures for training with duplicated signatures. In order to validate the correctness of our HMM implementation, we also tested the system on MCYT-100 using the first ten genuine signatures of each user for training and achieved an equal error rate (EER) of 0.80% for random forgeries and 3.76% for skilled forgeries. These results were indeed very similar to the results reported in [37] for MCYT, namely, 1.04% for random forgeries and 3.36% for skilled forgeries, which demonstrates the validity of our implementation.

V. SINGLE REFERENCE SIGNATURE SYSTEM SET UP

The SRSS needs to set up several variables. These variables are $(d_D, d_{t_0}, d_\mu, d_\sigma, d_{\theta_s}, d_{\theta_e})$ for both SW and TW methods. These values are optimized experimentally using the SUSIG Visual subcorpus database and the DTW-based verification.

These values are set by looking for the best tradeoff among the following three criteria.

1) *Optimum Performance of the SRSS:* The performance is measured in terms of EER[2] of the SRSS for different values of the variables and the number of duplicates. As is usual in forensic environments, the skilled forger test is prioritized over the random one.
2) *Minimum Computational Load:* This criterion is tied to the number of duplicates which increases the computational load due to the cost of both duplicating the signature and classifying. In the case of the DTW, the training computational load increases quadratically with the number of duplicates and the testing load increases linearly. The Manhattan-based and HMM systems have the same time relationships for the load during testing. The duplicates are used to find the optimal model parameters, which makes training take somewhat longer.
3) *Human-Like Duplicates:* To avoid duplicates beyond the natural intrapersonal variability, the images obtained are visually checked to limit the variability of the system parameters.

Once, the set up framework is established, the variables for the SW distortion method are obtained as follows.

1) The variables $(d_{t_0}, d_\mu, d_\sigma)$ are fixed to the values optimized in our preliminary study [10], i.e., $d_\mu = d_\sigma = 0.1$ and $d_{t_0} = 2.5$.
2) The search space for the remaining parameters $(d_D, d_{\theta_s}, d_{\theta_e})$ is simplified by applying the same deformation levels to each of them.
3) Table II shows the performance of the SRSS for a grid of $(d_D, d_{\theta_s}, d_{\theta_e})$ values and a different number of training duplicated signatures. It can be seen that there is a minimum in the error surface around 32 duplicates and $d_D = d_{\theta_s} = d_{\theta_e} = 0.2$ in the random forgery scenario. For a skilled forgery the minimum is reached around $d_D = d_{\theta_s} = d_{\theta_e} = 0.2$ and 128 duplicates.
4) Fig. 3 illustrates that this procedure generates human like signatures up to $d_D = d_{\theta_s} = d_{\theta_e} = 0.1$.
5) Looking for the minimum error in $d_D = d_{\theta_s} = d_{\theta_e} = 0.1$ columns and taking into account the goal of reducing the computational load, a tradeoff set up can be established in $d_D = d_{\theta_s} = d_{\theta_e} = 0.1$ and 32 duplicates.
6) A double check was performed at this point in order to analyze the duplicate stability. Thus, the SRSS was run ten times to obtain the following average performance and standard deviation: $(\overline{\text{EER}}_{\text{RF}} = 2.33\%, \sigma_{\text{RF}} = 0.12)$ and $(\overline{\text{EER}}_{\text{SF}} = 7.74\%, \sigma_{\text{SF}} = 0.20)$ for random and skilled forgery experiments, respectively.

For the TW distortion method, we followed the steps.

1) The parameters relating to the neuromuscular intrapersonal variability were fixed heuristically to $d_\mu = d_\sigma = 0.025$. Then, the search space was reduced to d_{t_0}.
2) Table III shows the performance of the SRSS for a grid of d_{t_0} values and a different number of training duplicates. In this case, the minimum error surface seems to be around 16 duplicates and $d_{t_0} = 0.05$ in the random forgery scenario and 64 duplicates and around $d_{t_0} = 0.05$ for the skilled forgery.

[2]The EER represents the operating point when the types I and II errors are coincident, i.e., false rejection ratio (FRR) and false acceptance ratio (FAR), respectively.

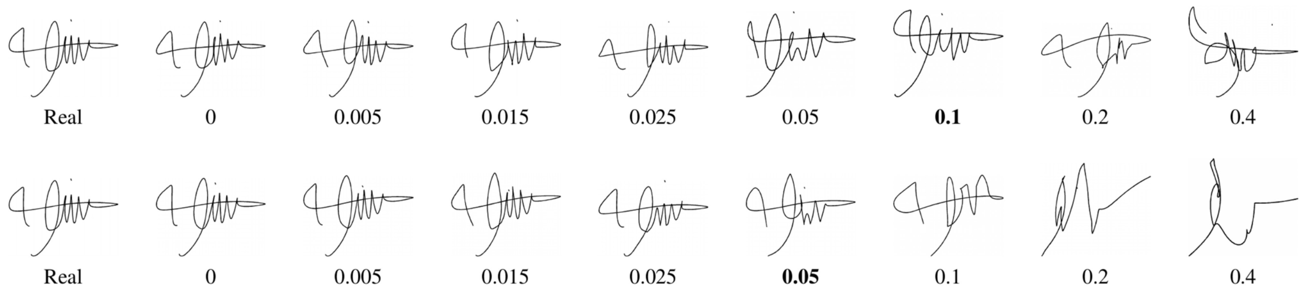

Fig. 3. Variation in the appearance of the signatures as a function of distortion increase. The first row refers to SW distortion method in which d_D, d_{θ_s}, and d_{θ_e} are changed; the second row corresponds to the TW distortion method, in which d_{t_0} is tuned.

TABLE III
SUSIG-VISUAL EER (%) RESULTS TRAINING WITH THE FIRST SIGNATURE PLUS DUPLICATES; TW DISTORTION METHOD AND DTW-BASED VERIFIER

Real	Dup.	Random Forgery Test								Skilled Forgery Test							
		d_{t_0}								d_{t_0}							
		0	0.005	0.015	0.025	0.05	0.1	0.2	0.4	0	0.005	0.015	0.025	0.05	0.1	0.2	0.4
1	0	8.09	8.09	8.09	8.09	8.09	8.09	8.09	8.09	15.53	15.53	15.53	15.53	15.53	15.53	15.53	15.53
1	2	8.60	7.45	6.26	4.68	3.62	4.98	4.51	6.68	11.81	11.91	12.13	12.66	9.26	8.72	10.11	10.11
1	4	7.32	7.36	5.91	4.13	2.72	3.83	4.77	8.26	11.28	10.74	11.17	13.40	8.30	7.77	9.26	9.79
1	8	6.55	7.49	4.34	3.79	2.17	3.45	4.72	10.85	9.79	10.85	10.11	14.15	7.66	7.45	8.30	9.47
1	16	5.45	6.13	4.17	3.40	1.45	3.40	5.96	12.26	9.79	9.79	9.68	14.36	7.34	7.23	7.98	8.72
1	32	5.36	5.74	3.70	3.40	1.49	3.23	6.94	13.11	9.68	9.68	8.30	13.51	7.02	7.13	8.30	8.62
1	64	5.19	5.15	3.53	3.45	**1.62**	3.40	7.57	14.38	10.11	9.36	8.09	12.02	**6.60**	6.81	8.09	8.72
1	128	4.98	4.68	3.83	3.40	1.74	3.40	7.02	15.53	10.21	9.26	8.19	11.06	6.60	6.60	8.09	8.62
1	256	4.72	8.43	3.57	3.23	2.21	3.49	8.43	16.26	9.89	7.77	10.00	16.91	6.47	6.70	7.77	8.62

3) The second row of Fig. 3 suggests that this second procedure generates human-like signatures up to $d_{t_0} = 0.05$.
4) Prioritizing the skilled forgery scenario over the random forgery as is usual in forensic environments, the set up can be stablished at $d_{t_0} = 0.05$ and 64 duplicates.
5) Running the complete system ten times at this operative point for a double check, we obtained ($\overline{\text{EER}}_{\text{RF}} = 1.55\,\%, \sigma_{\text{RF}} = 0.08$) and ($\overline{\text{EER}}_{\text{SF}} = 6.67\,\%, \sigma_{\text{SF}} = 0.09$) for random and skilled forgery experiments, respectively. These results confirm the stability of the selected operative points.

VI. EXPERIMENTAL RESULTS

The experimental evaluation aims to validate the SRSS in several ways. First, the human likeness of the duplicated signatures is assessed via a visual Turing test. Second, the reliability of the designed SRSS is tested through multiple public databases and standard verifiers. Finally, a performance comparison with the state-of-the-art systems positions this paper with respect to previous work that uses several samples for training.

A. Visual Turing Test Validation

The human capacity to distinguish our duplicated signatures from real specimens has been evaluated through a visual Turing test. This consists in measuring the human ability to distinguish between real and computer duplicated signatures. With this aim, a number of pairs of signatures were showed to different volunteers. In each pair, the first was the reference signature and the second was a duplicate of the given reference signature. Each volunteer was questioned about the authorship of the second one, i.e., whether the second one was duplicated by a human being or by a computer. In this one-by-one process, the reference signature was the same for each of five consecutive questions.

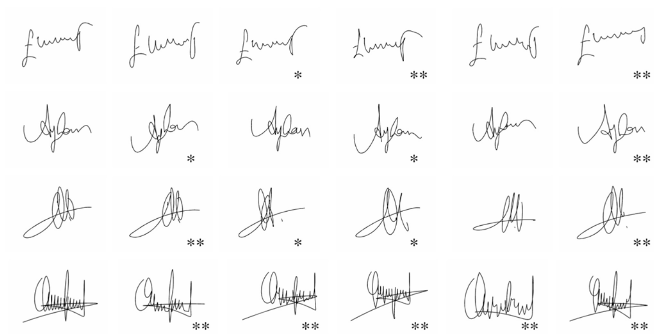

Fig. 4. Visual Turing test subset. The first column shows the reference signatures. The following columns show duplicated signatures. No asterisk means made by human beings, * means duplicated with the SW method, and ** means duplicated with the TW method.

Similarly to [15], [26], and [38], a set of 100 questioned specimens composed of 50 signatures written by real human beings, 25 duplicates following the SW method and 25 duplicates made on the basis of the TW method were judged by 100 nonforensic volunteers from several Western countries. Fig. 4 shows a subset of this experiment.

Once the questioning had been conducted, different measures were carried out to evaluate the distinguishability of

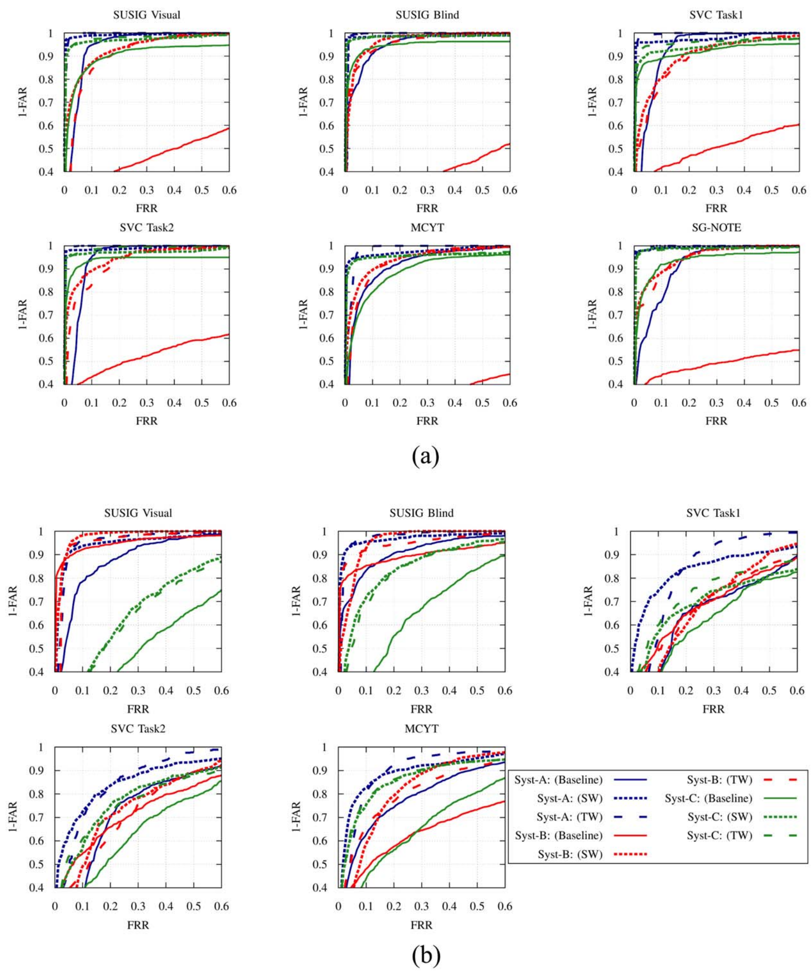

Fig. 5. ROC curves for (a) random and (b) skilled forgery tests using three verifiers and six databases. Systems A, B, and C are the DTW, Manhattan and HMM-based classifiers, respectively.

TABLE IV
VISUAL EXPERIMENT RESULTS

FSWR	FTWR	FRR	FDR	ACE
50.78 %	52.33 %	51.56 %	51.59 %	51.57 %
P = 0.826	P = 0.036	P = 0.098	P = 0.017	

FSWR *False Stroke-Wise Rate*: Error of misjudging a duplicated signature designed by the stroke-wise algorithm as real. **FTWR** *False Target-Wise Rate*: Error of misjudging a duplicated signature designed by the target-wise algorithm as real. **FRR** *False Real Rate*: The average between FSWR and FTWR. **FDR** *False Duplicated Rate*: The error made when a duplicated signature is judged as real. **ACE** *Average Classification Error*: Measured in global terms, the average between FRR and FDR

human and machine made duplicates. These measures are described in Table IV through different type of error rates. For all these error rates, 50% means that the real and duplicated signatures cannot be told apart.

These error rates are given in Table IV along with a binomial test indicating how probable the outcome is regarding a fair coin toss. For both SW and TW duplicates, the artificial signatures were considered as real in more than 50 % of the cases, which means that human beings were not able to distinguish duplicates from real signatures. For the TW duplicates (FTWR), the deviation from a fair coin toss was actually statistically significant ($P < 0.05$), i.e., human beings seem to be systematically biased toward judging the duplicates as real signatures. In summary, these results clearly emphasize the human likeness of the generated duplicates.

B. Performance Experiments

The results of the SRSS when tested against three standard automatic signature verifiers on six publicly available, online signature databases with the two duplication methods are illustrated at Table V. To establish a fair comparison, all verifiers had the same configuration for all databases and only the first registered signature per user was used for training in each case. The baselines were obtained by training with only one enrolled signature, without duplicates. For testing, we used all available genuine signatures from all remained users and forged signatures to compute the FAR curves for random and skilled forgery tests, respectively. In both experiments, all genuine signatures—except the first one—were used to estimate the FRR curves.

On the "System A: DTW-based," excellent results are obtained in the random forgery mode, the best being achieved with the SVC-Task2. This observation is reinforced by the skilled forgeries results, where we obtain the best results with the SUSIG-Blind subcorpus. It is also worth pointing out that

TABLE V
EER (%) COMPREHENSIVE EVALUATION USING THE FIRST ENROLLED REAL SIGNATURE PER USER FOR TRAINING. SW DENOTES THE STROKE-WISE DUPLICATION METHOD AND TW THE TARGET-WISE

Database	System A: DTW-based [7][24]						System B: Manhattan-based [33]						System C: HMM-based [28]					
	Random Forgery			Skilled Forgery			Random Forgery			Skilled Forgery			Random Forgery			Skilled Forgery		
	BL*	SW	TW	BL	SW	TW	BL	SW	TW	BL	SW	TW	BL	SW	TW	BL	SW	TW
SUSIG-Visual	8.09	2.13	1.62	15.53	7.45	6.60	46.85	11.36	12.64	8.51	5.53	5.85	11.98	4.76	4.32	40.96	30.64	31.60
SUSIG-Blind	9.45	1.91	1.54	13.75	5.68	5.22	52.14	8.05	8.86	13.64	8.52	8.64	7.19	2.86	2.76	31.25	18.07	18.52
SVC-Task1	10.50	4.00	1.50	29.13	17.25	17.88	44.00	13.60	15.20	29.50	27.88	28.25	10.79	8.16	5.53	33.25	27.00	24.12
SVC-Task2	8.10	1.90	0.50	23.66	18.25	18.63	42.50	10.40	12.80	28.00	25.00	27.88	7.50	3.81	3.68	31.88	22.38	23.88
MCYT100	12.48	5.04	4.04	23.20	13.72	13.56	56.32	10.20	10.96	33.88	20.36	21.36	14.62	5.79	5.66	31.96	16.32	16.24
Mobile	12.80	2.06	1.03	-	-	-	47.20	10.72	11.04	-	-	-	9.05	2.35	2.73	-	-	-

*BL means baseline

in this system, TW method performs slightly better than SW for all databases in random forgery and for three out of five in skilled forgeries.

On the "System B: Manhattan-based," relevant improvements are obtained in all cases for random forgeries. The most relevant effect is shown again in SUSIG-Blind subcorpus. Note that this database was not used to fine-tune the system, but only as a testing database. Moreover, we can observe that skilled forgery, the most difficult and relevant test, also improved on the baseline performance. Although the improvements are not as impressive as in random forgery, the performance is not impaired in any case. Additionally, although a comparison between SW and TW performances lead to obtaining similar findings, we can observe that the SW method achieves a better performance for the SVC-Task1 and SVC-Task2 databases.

On "System C: HMM-based," the potential of the designed SRSS is proven. All cases show notable improvements in random forgery. Although the TW only slightly improved the results, the best performance was given by the Mobile database with the SW method. In the case of skilled forgeries, the performance was also reduced for both duplication procedures, the TW method being a little better than for the SVC-Task1.

In general terms, experimental results highlight the robustness of the SRSS since the common tendency observed in all cases is that both duplication methods give improvements in all cases with respect to the baseline. Also, we could say that both duplication methods (SW and TW) perform in a similar way, thus highlighting a coherent improvement for both random and skilled forgery tests. Additionally, this is graphically illustrated in Fig. 5 with the ROC curves for both random and skilled forgery tests. So it is possible to conclude that both duplication methods mimic in a reasonable way the intrapersonal variability of the different databases which is the basic requirement for an efficient SRSS. The maximum impact on performance is provided by the DTW-based system.

C. Automatic Signature Verifier Results Comparison

To contextualize the SRSS with the state of the art, our experimental results obtained with the DTW-based verifier are compared in Table VI with state-of-the-art results for each of the six databases. The methods are ranked according to their reported performance in the skilled forgery test. The Table notes different experimental protocols which could introduce a certain bias.

On the SUSIG-Visual subcorpus, our result in the TW method for the skilled forgery test (EER = 6.67%) was very close to the results in [36] and [39] (4.37% and 5.38%, respectively). Both works were published only one year ago using five training signatures. For the random forgery test, only two methods achieved better results than ours: in [45] ten signatures were used for training and in [49] skilled forgeries along with six genuine specimens were added to the training set. These results highlight the potential of the proposed system. Note that the good results could be explained by the fact that our method was optimized with respect to this database.

On the SUSIG-Blind subcorpus, not many results have been reported in the literature. Our EER was 2.37 % higher than that reported in [29] for the skilled forgery test. Moreover, it was 1.28% lower than that in [29] for the random forgery test, highlighting the potential of a two-stage verifier.

On the MCYT corpus the comparison was biased among the different methods, mainly because of the number of users. We have chosen 100 users as with the majority of the methods. Although the number of users varies among the reported work, it can be seen that our proposal improves the EER for both tests considered with respect to the only other method that uses one training signature [23]. Regarding to the subsequent ranked method [40], the EER of our system was 5.76% lower for the skilled forgery test. For the random forgery test, our result was 1.94% lower than the closest systems [43] and [46].

On the SG-NOTE database, only the random forgery test was carried out because no forgeries were collected. Our system outperformed the reference system [32] by 0.80%. This is particularly interesting because the proposed SRSS was not optimized for this database. It reinforces the applicability of our method to mobile signatures on handheld devices.

On the SVC-Task1 subcorpus, only few results have been published. Despite differences in the training, our EER of 17.25% was relatively poor when compared with 3.16% in [49] and 2.84% in [30] for the skilled forgery test. The reference systems on the other hand, especially in the competition results, were optimized specifically for the SVC database. Nevertheless, better results were obtained again for the random forgery test, where the TW method with 1.50% of EER which outperformed the results in [30] with 1.85% and has performed just slightly worse when compared with EER = 0.45% in [49].

On the SVC-Task2 subcorpus, many methods have been studied in the literature, probably because it provides dynamic

TABLE VI
PERFORMANCE IN EER (%) FOR PUBLISHED VERIFICATION SYSTEMS

SUSIG-Visual			
Method	# Train	Random	Skilled
Our previous work [10]	1	3.61	7.87
This work, stroke-wise	**1**	**2.23**	**7.74**
This work, target-wise	**1**	**1.55**	**6.67**
Fuzzy modeling [39]	5	4.57	5.38
Histogram+Manhattan [36]	5	2.91	4.37
Pole-zero models [42]	5^a	1.97	3.91
FFT+DTW [40]	5	-	3.03
DCT+Sparse Repr. [45]	10	1.26	2.98
Stable Domain [47]	10	-	see^b
DTW+Linear C. [29]	5	4.08	2.10
Parzen window+DCT [49]	6^a	1.23	1.49
DTW+Linear C. [50]	5^c	-	1.40^f
Self-thought learning [52]	25 %^{d}	-	0.77

MCYT			
Method	# Train	Random	Skilled
Duplicated+HMM. [23] - u330	1	6.60	15.60
This work, stroke-wise - u100	**1**	**5.04**	**13.72**
This work, target-wise - u100	**1**	**4.04**	**13.56**
FFT+DTW [40] - u100	5	-	7.80
Normalization+Fractional [41] - u280	5	1.8	6.60
Symbolic Rep. [43] - u330	5	2.10	6.45
DTW-VQ [44] - u280	5	1.37	5.42
Time functions+LDP [46] - u100	5	2.10	5.20
Neuro-fuzzy system [48] - u100	5	-	4.88
Histogram+Manhattan [36] - u100	5	1.15	4.02
Function-based+HMM [37] - u145	10	1.04	3.36
Function-based+HMM [51] - u100	10	-	2.85

SUSIG-Blind			
Method	# Train	Random	Skilled
This work, stroke-wise	**1**	**1.91**	**5.68**
This work, target-wise	**1**	**1.54**	**5.22**
DTW+Linear C. [29]	5	2.82	2.85

SG-NOTE			
Method	# Train	Random	Skilled
Global features+Mahalanobis [32]	5	2.10	-
This work, stroke-wise	**1**	**2.06**	**-**
This work, target-wise	**1**	**1.03**	**-**

SVC-Task1			
Method	# Train	Random	Skilled
This work, target-wise	**1**	**1.50**	**17.88**
This work, stroke-wise	**1**	**4.00**	**17.25**
Parzen window+DCT [49]^e	6^a	0.49	3.61
SVC-competition [30]	5	1.85	2.84

SVC-Task2			
Method	# Train	Random	Skilled
This work, target-wise	**1**	**0.50**	**18.63**
This work, stroke-wise	**1**	**1.90**	**18.25**
Fuzzy modeling [39]	5	5.49	7.57
Function-based+HMM. [53]	5	1.06	7.14
LCCS-SVM. [54]	5	0.12	6.84
DCT+Sparse Repr. [45]	10	0.45	5.61
SVC-competition [30]	5	1.70	2.89
Parzen window+DCT [49]	6^a	0.37	2.04
Self-thought learning [52]	25 %^{d}	-	0.83

^{a} Skilled forgeries were added to the training set.
^{b} FRR:2.15; FAR:2.10.
^{c} 54 forgeries were added to train the classifier.
^{d} K-Fold Cross-Validation strategy.
^{e} Task1&2.
^{f} Error Rate.

pressure and pen orientation information. It should be noted that in our system we do not use this information because we designed the system to be applicable also to devices that do not capture such dynamic signals. This leads to a bias in the comparison in favor of the reference systems. As for the SVC-Task1, our system did not report results as competitive as other works: while our methods were able to achieve 18.25% in EER, the EER of the next lower ranked work [39] is 7.57%. Moreover, the EER for the random forgery test in the TW method is 1.50%, which is among the best results reported for this database. Apart from not using all available dynamic sequences of this database, the limited EER obtained in the skilled forgery test can be explained on the basis of this particular database. In this database the signers were used to signing with signatures composed of many components with few strokes and without flourishes. In contrast, the signatures in the other databases are composed of fewer components with many strokes per component and with flourishes.

To conclude this section, we see that our SRSS models properly reproduce the intrapersonal variability and the results were equivalent using more reference specimens in training. Although useful results were obtained in the experiments, improvements should be considered in the forgery scenario [55] in order to mimic the lack of 5–10 more reference signatures. In this case, multistroke signature generation among others could provide future improvement in the duplication methods.

VII. CONCLUSION

In this paper, we present a theoretical and experimental, novel SRSS that generates intrapersonal variability from the synthetic generation of duplicated signatures from only one signature. The duplicated specimens were produced from a neuromuscular model based on the kinematic theory of rapid human movements, and its sigma-lognormal parameters. This is one of the most mature models widely used in pattern analysis applications and verification systems. Two methods were presented to generate human-like duplicated signatures: the first is based on SW distortion, whereas the second pursues a TW distortion, directly applied to the position and orientation of the action plan. Experimental results have revealed that these approaches generate duplicated samples which are indistinguishable, as is demonstrated by a visual Turing test. Also, a performance-based test studied the behavior of the SRSS on multiple public databases and several state-of-the-art automatic signature verifiers. Our results suggest that, for the random forgery test, our system performs similarly to methods that use five reference signatures. Furthermore, our system achieves a slightly better performance in the skilled forgery test when compared with the analyzed state-of-the-art systems which use more than one reference sample. Although our results were competitive, to achieve a more accurate estimation of the intrapersonal variability in order to cope with skilled forgeries, more research is needed.

The future direction of this research follows up the study of the real stroke variability under the sigma-lognormal model. During signature execution the strokes are in general not consistent across several genuine signatures. A stability study [56] may be required to detect and align the stable strokes. As such, this alignment would allow the investigation of the real variability of the sigma-lognormal parameters per stroke and per signer. Duplication methods can attract further interest in automatic signature verifiers schemes which use more than one reference signature to train. A proper configuration of proposed methods could lead to accuracy improvements.

Finally, this novel framework opens the door to new competitions on signature verification using a single signature as reference. Also, it leads to work on signatures registered in several scenarios such as Wacom-like or handheld devices, where the dynamic features and the precision of the frame rate is not as accurate as the signatures from tablets or LCD devices [32]. This suggests the possibility of using handheld tablets for personal authentication in e-security problems, where only one reference sample is available.

Acknowledgment

This paper was initiated in the Scribens Laboratory at École Polytechnique de Montréal, followed by a subsequent visit to the University of Fribourg, Switzerland, and finalized at the University of Las Palmas de Gran Canaria, Spain. Therefore, M. Diaz wishes to thank Prof. R. Plamondon and Dr. A. Fischer, co-authors of this paper, for hosting him in their laboratories during the winter of 2014–2015 and the summer of 2015.

References

[1] H. Yan, J. Lu, and X. Zhou, "Prototype-based discriminative feature learning for kinship verification," *IEEE Trans. Cybern.*, vol. 45, no. 11, pp. 2535–2545, Nov. 2015.

[2] M. H. Lim and P. C. Yuen, "Entropy measurement for biometric verification systems," *IEEE Trans. Cybern.*, vol. 46, no. 5, pp. 1065–1077, May 2016.

[3] R. Plamondon, "A kinematic theory of rapid human movements: Part I: Movement representation and generation," *Biol. Cybern.*, vol. 72, no. 4, pp. 295–307, 1995.

[4] R. Plamondon, "A kinematic theory of rapid human movements: Part II: Movement time and control," *Biol. Cybern.*, vol. 72, no. 4, pp. 309–320, 1995.

[5] R. Plamondon and M. Djioua, "A multi-level representation paradigm for handwriting stroke generation," *Human Movement Sci.*, vol. 25, nos. 4–5, pp. 586–607, 2006.

[6] S. Djeziri, W. Guerfali, R. Plamondon, and J. M. Robert, "Learning handwriting with pen-based systems: Computational issues," *Pattern Recognit.*, vol. 35, no. 5, pp. 1049–1057, 2002.

[7] C. O'Reilly and R. Plamondon, "Impact of the principal stroke risk factors on human movements," *Human Movement Sci.*, vol. 30, no. 4, pp. 792–806, 2011.

[8] R. Plamondon, C. O'Reilly, J. Galbally, A. Almaksour, and É. Anquetil, "Recent developments in the study of rapid human movements with the kinematic theory: Applications to handwriting and signature synthesis," *Pattern Recognit. Lett.*, vol. 35, pp. 225–235, Jan. 2014.

[9] C. O'Reilly and R. Plamondon, "Development of a sigma-lognormal representation for on-line signatures," *Pattern Recognit.*, vol. 42, no. 12, pp. 3324–3337, 2009.

[10] M. Diaz, A. Fischer, R. Plamondon, and M. A. Ferrer, "Towards an automatic on-line signature verifier using only one reference per signer," in *Int. Conf. Document Anal. Recognit. (ICDAR)*, Tunis, Tunisia, 2015, pp. 631–635.

[11] A. Fischer and R. Plamondon, "A dissimilarity measure for on-line signature verification based on the sigma-lognormal model," in *Proc. Biennial Conf. Int. Graphonomics Soc.*, 2015, pp. 83–86.

[12] M. Gomez-Barrero, J. Galbally, J. Fierrez, J. Ortega-Garcia, and R. Plamondon, "Enhanced on-line signature verification based on skilled forgery detection using sigma-lognormal features," in *Proc. Conf. Biometrics (ICB)*, 2015, pp. 501–506.

[13] M. Gomez-Barrero, J. Galbally, R. Plamondon, J. Fierrez, and J. Ortega-Garcia, "Variations of handwritten signatures with time: A sigma-lognormal analysis," in *Proc. Conf. Biometrics (ICB)*, Madrid, Spain, 2013, pp. 1–6.

[14] J. Galbally, R. Plamondon, J. Fierrez, and J. Ortega-Garcia, "Synthetic on-line signature generation. Part I: Methodology and algorithms," *Pattern Recognit.*, vol. 45, no. 7, pp. 2610–2621, 2012.

[15] J. Galbally, J. Fierrez, J. Ortega-Garcia, and R. Plamondon, "Synthetic on-line signature generation. Part II: Experimental validation," *Pattern Recognit.*, vol. 45, no. 7, pp. 2622–2632, 2012.

[16] R. Plamondon and G. Lorette, "Automatic signature verification and writer identification—The state of the art," *Pattern Recognit.*, vol. 22, no. 2, pp. 107–131, 1989.

[17] F. Leclerc and R. Plamondon, "Automatic signature verification: The state of the art—1989–1993," *Int. J. Pattern Recognit. Artif. Intell.*, vol. 8, no. 3, pp. 643–660, 1994.

[18] M. C. Fairhurst, "Signature verification revisited: Promoting practical exploitation of biometric technology," *Electron. Commun. Eng. J.*, vol. 9, no. 6, pp. 273–280, Dec. 1997.

[19] R. Plamondon and S. N. Srihari, "Online and off-line handwriting recognition: A comprehensive survey," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 22, no. 1, pp. 63–84, Jan. 2000.

[20] D. Impedovo and G. Pirlo, "Automatic signature verification: The state of the art," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 38, no. 5, pp. 609–635, Sep. 2008.

[21] D. Impedovo, G. Pirlo, and R. Plamondon, "Handwritten signature verification: New advancements and open issues," in *Proc. Int. Conf. Frontiers Handwriting Recognit.*, Bari, Italy, 2012, pp. 367–372.

[22] V. L. Blankers, C. E. van den Heuvel, K. Y. Franke, and L. G. Vuurpijl, "ICDAR 2009 signature verification competition," in *Int. Conf. Doc. Anal. Recognit. (ICDAR)*, Barcelona, Spain, 2009, pp. 1403–1407.

[23] J. Galbally, J. Fiérrez, M. Martinez-Diaz, and J. Ortega-Garcia, "Improving the enrollment in dynamic signature verfication with synthetic samples," in *Int. Conf. Doc. Anal. Recognit. (ICDAR)*, Barcelona, Spain, 2009, pp. 1295–1299.

[24] C. Rabasse, R. M. Guest, and M. C. Fairhurst, "A new method for the synthesis of signature data with natural variability," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 38, no. 3, pp. 691–699, Jun. 2008.

[25] M. Diaz-Cabrera, M. A. Ferrer, and A. Morales, "Cognitive inspired model to generate duplicated static signature images," in *Proc. Int. Conf. Frontiers Handwriting Recognit. (ICFHR)*, Heraklion, Greece, 2014, pp. 61–66.

[26] M. A. Ferrer, M. Diaz-Cabrera, and A. Morales, "Static signature synthesis: A neuromotor inspired approach for biometrics," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 37, no. 3, pp. 667–680, Mar. 2015.

[27] R. Plamondon and F. J. Maarse, "An evaluation of motor models of handwriting," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 19, no. 5, pp. 1060–1072, Sep./Oct. 1989.

[28] C. O'Reilly and R. Plamondon, "A globally optimal estimator for the delta-lognormal modeling of fast reaching movements," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 5, pp. 1428–1442, Oct. 2012.

[29] A. Kholmatov and B. Yanikoglu, "SUSIG: An on-line signature database, associated protocols and benchmark results," *Pattern Anal. Appl.*, vol. 12, no. 3, pp. 227–236, 2009.

[30] D.-Y. Yeung *et al.* "SVC2004: First international signature verification competition," in *Proc. 1st Int. Conf. Biometric Authentication (ICBA)*, vol. 3072. 2004, pp. 16–22.

[31] J. Ortega-Garcia *et al.* "MCYT baseline corpus: A bimodal biometric database," *IEE Proc. Vis. Image Signal Process.*, vol. 150, no. 6, pp. 395–401, Dec. 2003.

[32] M. Martinez-Diaz, J. Fierrez, R. P. Krish, and J. Galbally, "Mobile signature verification: Feature robustness and performance comparison," *IET Biometrics*, vol. 3, no. 4, pp. 267–277, Dec. 2014.

[33] M. Djioua and R. Plamondon, "A new algorithm and system for the characterization of handwriting strokes with delta-lognormal parameters," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 31, no. 11, pp. 2060–2072, Nov. 2009.
[34] A. Fischer, M. Diaz, R. Plamondon, and M. A. Ferrer, "Robust score normalization for DTW-based on-line signature verification," in *Proc. Int. Conf. Doc. Anal. Recognit. (ICDAR)*, Tunis, Tunisia, 2015, pp. 241–245.
[35] H. Sakoe and S. Chiba, "Dynamic programming algorithm optimization for spoken word recognition," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 26, no. 1, pp. 43–49, Feb. 1978.
[36] N. Sae-Bae and N. Memon, "Online signature verification on mobile devices," *IEEE Trans. Inf. Forensics Security*, vol. 9, no. 6, pp. 933–947, Jun. 2014.
[37] J. Fierrez, J. Ortega-Garcia, D. Ramos, and J. Gonzalez-Rodriguez, "HMM-based on-line signature verification: Feature extraction and signature modeling," *Pattern Recognit. Lett.*, vol. 28, no. 16, pp. 2325–2334, 2007.
[38] B. M. Lake, R. Salakhutdinov, and J. B. Tenenbaum, "Human-level concept learning through probabilistic program induction," *Science*, vol. 350, no. 6266, pp. 1332–1338, 2015.
[39] A. Q. Ansari, M. Hanmandlu, J. Kour, and A. Singh, "Online signature verification using segment-level fuzzy modelling," *IET Biometrics*, vol. 3, no. 3, pp. 113–127, Sep. 2014.
[40] B. A. Yanikoglu and A. Kholmatov, "Online signature verification using Fourier descriptors," *EURASIP J. Adv. Signal Process.*, vol. 2009, Jan. 2009, Art. no. 12.
[41] C. Vivaracho-Pascual, M. Faundez-Zanuy, and J. M. Pascual, "An efficient low cost approach for on-line signature recognition based on length normalization and fractional distances," *Pattern Recognit.*, vol. 42, no. 1, pp. 183–193, 2009.
[42] S. Rashidi, A. Fallah, and F. Towhidkhah, "Authentication based on pole-zero models of signature velocity," *J. Med. Signals Sensors*, vol. 3, no. 4, pp. 195–208, 2013.
[43] D. S. Guru and H. N. Prakash, "Online signature verification and recognition: An approach based on symbolic representation," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 31, no. 6, pp. 1059–1073, Jun. 2009.
[44] M. Faundez-Zanuy, "On-line signature recognition based on VQ-DTW," *Pattern Recognit.*, vol. 40, no. 3, pp. 981–992, 2007.
[45] Y. Liu, Z. Yang, and L. Yang, "Online signature verification based on DCT and sparse representation," *IEEE Trans. Cybern.*, vol. 45, no. 11, pp. 2498–2511, Nov. 2015.
[46] L. Nanni and A. Lumini, "A novel local on-line signature verification system," *Pattern Recognit. Lett.*, vol. 29, no. 5, pp. 559–568, 2008.
[47] G. Pirlo, V. Cuccovillo, M. Diaz-Cabrera, D. Impedovo, and P. Mignone, "Multidomain verification of dynamic signatures using local stability analysis," *IEEE Trans. Human-Mach. Syst.*, vol. 45, no. 6, pp. 805–810, Dec. 2015.
[48] K. Cpałka , M. Zalasiński, and L. Rutkowski, "A new algorithm for identity verification based on the analysis of a handwritten dynamic signature," *Appl. Soft Comput.*, vol. 43, pp. 47–56, Jun. 2016.
[49] S. Rashidi, A. Fallah, and F. Towhidkhah, "Feature extraction based DCT on dynamic signature verification," *Scientia Iranica*, vol. 19, no. 6, pp. 1810–1819, 2012.
[50] A. Kholmatov and B. Yanikoglu, "Identity authentication using improved online signature verification method," *Pattern Recognit. Lett.*, vol. 26, no. 15, pp. 2400–2408, 2005.
[51] E. A. Rua and J. L. A. Castro, "Online signature verification based on generative models," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 4, pp. 1231–1242, Aug. 2012.
[52] M. Fayyaz, M. H. Saffar, M. Sabokrou, and M. Fathy, "Feature representation for online signature verification," *CoRR abs/1505.08153*, 2015.
[53] J. Fierrez-Aguilar, J. Ortega-Garcia, and J. Gonzalez-Rodriguez, "Target dependent score normalization techniques and their application to signature verification," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 35, no. 3, pp. 418–425, Aug. 2005.
[54] C. Gruber, T. Gruber, S. Krinninger, and B. Sick, "Online signature verification with support vector machines based on LCSS kernel functions," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 40, no. 4, pp. 1088–1100, Aug. 2010.
[55] J. J. Brault and R. Plamondon, "A complexity measure of handwritten curves: Modeling of dynamic signature forgery," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 23, no. 2, pp. 400–413, Mar./Apr. 1993.
[56] A. Parziale, S. G. Fuschetto, and A. Marcelli, "Modeling stability in on-line signatures," *J. Forensic Doc. Examiners*, vol. 24, pp. 37–45, Dec. 2014.

Moises Diaz (M'15) received the two M.Tech., M.Sc., and Ph.D. degrees in engineering, and the M.Ed. degree in secondary education from La Universidad de Las Palmas de Gran Canaria, Las Palmas, Spain, in 2010, 2011, 2016, and 2013, respectively.

He has been a visiting student with the University of Salerno, Fisciano, Italy, the University of Bari, Bari, Italy, the University of Parma, Parma, Italy, the University of Fribourg, Fribourg, Switzerland, Indian Statistical Institute, Kolkata, India, and the École Polytechnique de Montréal, Montreal, QC, Canada. His current research interests include pattern recognition, document analysis, handwriting recognition, biometrics, computer vision, and intelligent transportation systems.

Dr. Diaz is a member of the IAPR.

Andreas Fischer (M'15) received the M.S. and Ph.D. degrees in computer science from the University of Bern, Bern, Switzerland, in 2008 and 2012, respectively.

He was a Post-Doctoral Researcher with the Centre for Pattern Recognition and Machine Intelligence and the École Polytechnique, Montreal, QC, Canada. Since 2015, he has been a Senior Researcher and a Lecturer with the University of Fribourg, Switzerland, and the University of Applied Sciences and Arts Western Switzerland, Fribourg, Switzerland. His current research interests include historical document analysis, handwriting recognition, keyword spotting, signature verification, hidden Markov models, recurrent neural networks, and graph-based pattern recognition.

Miguel A. Ferrer received the M.Sc. and Ph.D. degrees from the Universidad Politécnica de Madrid, Madrid, Spain, in 1988 and 1994, respectively.

He joined the University of Las Palmas de Gran Canaria, Las Palmas, Spain, in 1989, where he is currently a Full Professor. He established the Digital Signal Processing Research Group in 1990. His current research interests include pattern recognition, biometrics, audio quality, and computer vision applications to fisheries and aquaculture.

Réjean Plamondon (F'00) received the B.Sc. degree in physics and the M.Sc.A. and Ph.D. degrees in electrical engineering from Université Laval, Quebec City, QC, Canada, in 1973, 1975, and 1978, respectively.

In 1978, he joined the faculty of the École Polytechnique, Université de Montréal, Montreal, QC, Canada, where he is currently a Full Professor. He has been the Head of the Department of Electrical and Computer Engineering from 1996 to 1998 and the President of École Polytechnique from 1998 to 2002, where he is currently the Head of the Laboratoire Scribens. He has been involved in many pattern recognition projects, particularly in the field of on-line and off-line handwriting analysis and processing. His current research interests include development of a kinematic theory of rapid human movements which can take into account, with the help of lognormal functions, the major psychophysical phenomena reported in studies dealing with rapid movements. The theory has been found successful in describing the basic kinematic properties of velocity profiles as observed in finger, hand, arm, head and eye movements. He has studied and analyzed these biosignals extensively in order to develop creative and powerful methods and systems in various domains of engineering.

Prof. Plamondon was a recipient of the International Association for Pattern Recognition (IAPR)/ICDAR 2013 Outstanding Achievement Award for "Theoretical contributions to the understanding of human movement and its applications to signature verification, handwriting recognition, instruction, and health assessment, and for promoting online document processing in numerous multidisciplinary fields." He is a fellow of The Netherlands Institute for Advanced Study in the Humanities and Social Sciences in 1989 and the IAPR in 1994.